

# Improving Tail Compatibility Through Sequential Distributed Model Predictive Control

Yanhao Yang, Joseph Norby, Justin K. Yim, and Aaron M. Johnson

Department of Mechanical Engineering

Carnegie Mellon University

Pittsburgh, PA, USA

Email: yanhaoy@andrew.cmu.edu

**Abstract**—Model predictive control (MPC) is an effective strategy for controlling constrained dynamical systems such as mobile robots. However, when actuation is distributed into distinct subsystems such as legs and tails, it is often undesirable to require this particular controller form for all such subsystems or to isolate each component entirely. In this work, we present a sequential distributed MPC for tailed, legged robots that separates the controllers of the different components while retaining communication of task information between them. This scheme improves the versatility of the tail, allows the reuse of existing leg controllers, and reduces the required information between the components. Simulation results show that when a quadrupedal robot unexpectedly misses foot contact, the sequential distributed control scheme can retain over 90% of the disturbance rejection performance of the centralized controller while requiring only a prediction of the net moment on the body.

## I. INTRODUCTION

The challenge of controlling legged robots comes from the intermittent foot contact, but many recent legged robots have developed strategies to mitigate this challenge and shown excellent locomotion performance in relatively smooth terrains [11, 8, 6, 7, 9]. However, this paradigm can be susceptible to failure on rough terrain due to missed foot positions or slippage on rocky hills. We believe that the biologically inspired tails can help control body orientation and maintaining balance to counteract these effects [12].

In order to realize the potential utility of the tail, we need a controller that can handle fast-changing, highly under-actuated, and constrained dynamical systems which are caused by insufficient foot contact force to balance the body. This controller should have the following properties: 1) update control commands online to account for unstructured environments and uncertain foot contact failure, 2) have a sufficient prediction horizon to quickly identify future failures, 3) capture the time-varying and non-linear dynamics of tailed actuation, and 4) navigate kinematic constraints such as avoiding the tail contacting the body and limiting the range of tail motion

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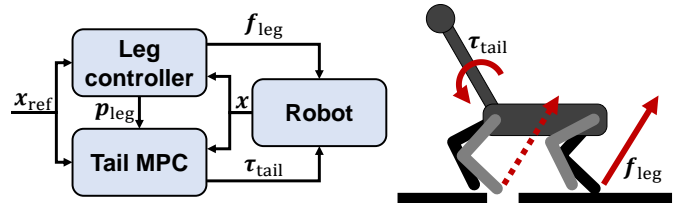


Fig. 1. Left: Proposed sequential distributed MPC scheme. According to the sensor measurements  $x$  and reference inputs  $x_{\text{ref}}$ , the leg controller calculates the GRF  $f_{\text{leg}}$  and passes information  $p_{\text{leg}}$  for approximating the net moment of leg actuation to the tail controller solves the tail motor torque  $\tau_{\text{tail}}$  to balance the robot. Right: the robot misses the contact of the right rear foot, uses its tail to maintain balance, and waits for the next gait cycle to resume the support of both feet.

before falling to the ground to prevent the tail from breaking. For these reasons, MPC is an ideal control strategy for the tail. [4] has demonstrated the effectiveness of MPC in actuating legs and a tail together to reject lateral disturbances in a legged robot.

However, requiring the leg controller to match that of the tail has disadvantages. Including the tail within the leg controller increases the size of the system and the complexity of the multi-body dynamics. Moreover, forcing the legs and tail to use the same controller may be undesirable, as many non-MPC leg controllers have demonstrated remarkable success, e.g. [10]. Since the tail mainly affects the orientation of the body while the legs can control translation and orientation, it is quite reasonable to decouple their control as in [3]. Prior work in distributed MPC has exploited this type of structure by separating the control problem into subproblems and solving them in various orders [2].

In this abstract, we present a sequential distributed scheme of MPC for tailed robots, summarized in Fig. 1, which allows the tail to use only a small amount of information to account for the effect of leg actuation without prescribing a particular form of leg controller. We apply this method to control a 2-DOF tail to balance a robot subject to an unexpected loss of foot contact and show that it has minimal performance loss compared to a centralized MPC that includes all components in the same optimization.

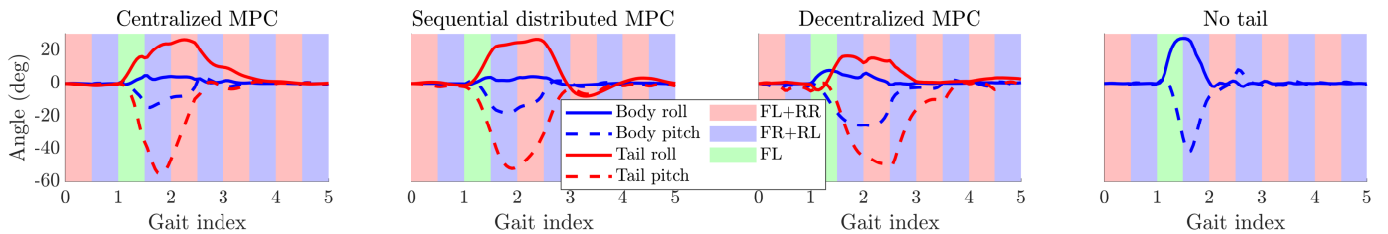


Fig. 2. Body roll (blue solid), body pitch (blue dotted), tail roll (red solid), and tail pitch (red dotted) when a foot misses stance. Shading indicates support by the left front leg and right rear leg (red shaded), by the right front leg and left rear leg (blue shaded), and only by the left front leg (green shaded).

TABLE I  
COMPARISON OF DIFFERENT CONTROL SCHEMES

Method	Max angle error (deg)	Arbitrary leg controller
Centralized MPC	15.1	No
Sequential distributed MPC	18.0	Yes
Decentralized MPC	26.1	Yes
No tail	48.4	Yes

## II. METHODS

As shown at the left of Fig. 1, the sequential distributed control scheme consists of three stages. First, the leg controller solves for the leg control commands. Second, key leg information is extracted from these commands, in particular a prediction of the net moment applied to the body over time. Third, this information is input into the tail MPC to solve a nonlinear program (NLP) to balance the robot and obtain the tail controls. The leg controller implemented in this work adopts a centroidal dynamics MPC to solve for the ground reaction force (GRF) which is fed into the tail controller. Note that this leg controller could be replaced with any existing controller, as long as the approximate net moment on the body is known. The required leg information could be GRFs, the contact sequence, or joint torque commands. In this way, we can reuse existing leg controllers, reduce the information required for the tail, and achieve asynchronous parallel control of the tail and legs.

In order to evaluate the performance of the sequential distributed MPC scheme, we consider a foot contact failure scenario, as shown at the right of Fig. 1. The robot is simplified into a single rigid body and the 2-DOF tail is approximated as a point mass connected to the back of the body through serial roll and pitch motors. The ratios of the tail inertia to the body inertia in the roll, pitch, and yaw axes are 2.37, 0.63, and 0.56 respectively. The leg input is modeled as the GRF applied at the foot position, and the tail input consists of the torque of two motors. The robot is commanded to trot in a straight line at a speed of 2.5 body lengths per second with a gait period of 0.5s. The foot positions are planned according to Raibert's heuristic [10]. In the first half of the second cycle, the rear right foot misses its position, resulting in no GRF on the foot and thus highly underactuated dynamics. We make the assumption that the robot detects this event immediately and is able to resume double support when the next foot touches down. At the same

time, the other feet maintain the planned foot positions. The cost function for the tail control is composed of weighted body orientation error and tail swing amplitude. System dynamics are modeled in CasADi [1], the NLP is solved by IPOPT [13], and the dynamics are integrated with CVODES [5] to yield state trajectories.

In the simulation, we compared the performance of different strategies, namely fully centralized MPC, sequential distributed MPC (GRF as leg information), and fully decentralized MPC (no information exchange). We simulate this scenario with no tail as a reference for evaluation. Fig. 2 show the roll and pitch trajectories of the body and tail. Table I lists the maximum axis-angle error of the body orientation and the requirement on the leg controller.

## III. RESULTS AND DISCUSSION

The results show that with proper leg information such as GRF, the sequential distributed control can maintain 91.3% of the performance (reduction in orientation error) of the centralized control, while only 67.0% for the decentralized one. Note that in the case of decentralized control lacking leg information, the tail response is significantly delayed compared to the body. This is because the tail controller cannot predict body motion in this case, so its behavior is similar to a feedback controller. In general, the results show that the tail with sequential distributed MPC has good compatibility with existing leg controllers (such as centroidal dynamics MPC).

Future work will improve the accuracy of net moment prediction and reduce the amount of information required to be compatible with existing leg controllers. Secondly, we will consider how to adapt MPC to handle less predictable leg controllers such as feedback controllers. More generally, it is still an open question of how to best use this communication to allow different components to collaborate more intelligently. For example, the legs could activate the tail when leg actuation alone cannot balance the body but otherwise allow the tail to return to its nominal configuration. Similarly, when the tasks of the subsystems conflict, such as when the bounding task of the leg controller does not match the balance task of the tail controller, the distributed communication network should be able to perform arbitration. Finally, the distributed MPC communication network between leg and tail controllers could be expanded to introduce two-way or iterative communication.

# REFERENCES

- [1] Joel A E Andersson, Joris Gillis, Greg Horn, James B Rawlings, and Moritz Diehl. CasADi – A software framework for nonlinear optimization and optimal control. *Mathematical Programming Computation*, 11(1): 1–36, 2019. doi: 10.1007/s12532-018-0139-4.
- [2] Panagiotis D. Christofides, Riccardo Scattolini, David Muñoz de la Peña, and Jinfeng Liu. Distributed model predictive control: A tutorial review and future research directions. *Computers & Chemical Engineering*, 51:21–41, 2013. ISSN 0098-1354. doi: <https://doi.org/10.1016/j.compchemeng.2012.05.011>. URL <https://www.sciencedirect.com/science/article/pii/S0098135412001573>. CPC VIII.
- [3] Avik De and Daniel E. Koditschek. Parallel composition of templates for tail-energized planar hopping. In *2015 IEEE International Conference on Robotics and Automation (ICRA)*, pages 4562–4569, 2015. doi: 10.1109/ICRA.2015.7139831.
- [4] Randall T. Fawcett, Abhishek Pandala, Jeeseop Kim, and Kaveh Akbari Hamed. Real-time planning and nonlinear control for quadrupedal locomotion with articulated tails. *Journal of Dynamic Systems, Measurement, and Control*, 143(7), Feb 2021. ISSN 0022-0434. doi: 10.1115/1.4049555. URL <https://doi.org/10.1115/1.4049555>. 071004.
- [5] Alan C Hindmarsh, Peter N Brown, Keith E Grant, Steven L Lee, Radu Serban, Dan E Shumaker, and Carol S Woodward. SUNDIALS: Suite of nonlinear and differential/algebraic equation solvers. *ACM Transactions on Mathematical Software (TOMS)*, 31(3):363–396, 2005.
- [6] M. Hutter, C. Gehring, A. Lauber, F. Gunther, C. D. Bellicoso, V. Tsounis, P. Fankhauser, R. Diethelm, S. Bachmann, M. Bloesch, H. Kolvenbach, M. Bjelonic, L. Isler, and K. Meyer. ANYmal - toward legged robots for harsh environments. *Advanced Robotics*, 31(17):918–931, 2017. doi: 10.1080/01691864.2017.1378591. URL <https://doi.org/10.1080/01691864.2017.1378591>.
- [7] Benjamin Katz, Jared Di Carlo, and Sangbae Kim. Mini Cheetah: A platform for pushing the limits of dynamic quadruped control. In *2019 International Conference on Robotics and Automation (ICRA)*, pages 6295–6301, 2019. doi: 10.1109/ICRA.2019.8793865.
- [8] Gavin Kenneally, Avik De, and D. E. Koditschek. Design principles for a family of direct-drive legged robots. *IEEE Robotics and Automation Letters*, 1(2):900–907, 2016. doi: 10.1109/LRA.2016.2528294.
- [9] Marc Raibert, Kevin Blankespoor, Gabriel Nelson, and Rob Playter. BigDog, the rough-terrain quadruped robot. *IFAC Proceedings Volumes*, 41(2):10822–10825, 2008. ISSN 1474-6670. doi: <https://doi.org/10.3182/20080706-5-KR-1001.01833>. URL <https://www.sciencedirect.com/science/article/pii/S1474667016407020>. 17th IFAC World Congress.
- [10] Marc H. Raibert. *Legged Robots That Balance*. Massachusetts Institute of Technology, USA, 1986. ISBN 0262181177.
- [11] C Semini, N G Tsagarakis, E Guglielmino, M Focchi, F Cannella, and D G Caldwell. Design of HyQ – a hydraulically and electrically actuated quadruped robot. *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*, 225(6):831–849, 2011. doi: 10.1177/0959651811402275. URL <https://doi.org/10.1177/0959651811402275>.
- [12] Stacey Shield, Ricardo Jericevich, Amir Patel, and Ardian Jusufi. Tails, Flails and Sails: How Appendages Improve Terrestrial Maneuverability by Improving Stability. *Integrative and Comparative Biology*, 05 2021. ISSN 1540-7063. doi: 10.1093/icb/icab108. URL <https://doi.org/10.1093/icb/icab108>. icab108.
- [13] Andreas Wächter and Lorenz T. Biegler. On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming. *Mathematical Programming*, 106(1):25–57, Mar 2006. ISSN 1436-4646. doi: 10.1007/s10107-004-0559-y. URL <https://doi.org/10.1007/s10107-004-0559-y>.